Effects of Heuristic Method of Teaching on Students' Achievement in Algebra

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Abstract - This study investigated the effects of heuristic method of teaching on students' achievement in algebra. The study employed a non equivalent control group quasi experimental design. The study was carried out in Anambra State of Nigeria. Two schools were used for the study. The treatment group was taught topics in theory of indices using heuristic method while control group was taught the same topics using Conventional Approach. At the onset of the experiment pre-test (Algebra Achievement Test) was administered to the students in the two groups while at the end of the treatment session that lasted eight weeks the same Algebra Achievement Test was administered to the students as post-test.. Three research questions and three Null hypotheses guided the study. The research questions were answered using mean and standard deviation while the Null hypotheses were tested at an alpha level of 0.05 using Analysis of Covariance (ANCOVA). The Results of the study show that the heuristic approach is superior to the conventional approach in fostering students' achievement in linear algebra. The study further revealed that male students taught linear algebra using the heuristic method performed better than their female counterparts and that there is no interaction between method and gender on students' achievement in linear algebra.

Key words: Heuristic, algebra, achievement, cognitive, intact classes, problem solving, pedagogy, quasi-experiment, science and technology

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1.1 Introduction

Heuristic, a word derived from the Greek language for "find" or "discover" is an adjective for experience-based techniques that help in problem solving, learning and discovery (Katarzyna and Jaszczolt, 2006). A heuristic method is particularly used to rapidly come to a solution that is hoped to be close to the best possible answer, or 'optimal solution'. Heuristics are "rules of thumb", educated guesses, intuitive judgments or simply common sense. It is a general way of solving problem. "Heuristics" as a noun is another name for heuristic methods. In more precise terms, heuristics stand for strategies using readily accessible, though loosely applicable, information to control problem solving in human beings and machines (Judea, 1983). It may be argued that the most fundamental heuristic is "trial and error", which can be used in everything from matching bolts to bicycles to finding the values of variables in algebraic problems.

In psychology, heuristics are simple, efficient rules, hard-coded by evolutionary processes or learned, which have been proposed to explain how people make decisions, come to judgments, and solve problems, typically when facing complex problems or incomplete information. These rules work well under most circumstances, but in extreme cases lead to systematic errors or cognitive biases (Kahneman, Tversky and Slovic, 1982). Although much of the work on discovering heuristics have been done by earlier psychologists, the novel application of the concept was originally introduced by Nobel laureate, Herbert Simon Gerd Gigerenzer, who focused on how heuristics can be used to make judgments that are in principle accurate, rather than producing cognitive biases (Gigerenzer and Todd, 1999).

In 2002, Daniel Kahneman and Shane Fredrick proposed that cognitive heuristics work by a process called "attribute substitution" which happens without conscious awareness. According to this theory, when someone makes judgments (of a "target attribute) that is computationally complex, they instead substitute a more easily calculated "heuristic attribute". In effect, they deal with a cognitively difficult problem by answering a simpler problem, without being aware that this is happening. This theory explains cases where judgments fail to show regression toward the mean (Kahneman 2003). In philosophy, especially in Continental European philosophy, the adjective "heuristic" (or the designation "heuristic device") is used when an entity X exists to enable understanding of, or knowledge concerning, some other entity Y. A classic example is the notion of utopia as described in Plato's best-known work, "Plato's Republic or the Republic" (Khnaemen, 2003). This means that the "ideal city" as depicted in the "The Republic" is not given as something to be pursued, or to present an orientation-point for development; rather, it shows how things would have to be connected, and how one thing would lead to another (often with highly problematic results), if one would opt for certain principles and carry them through rigorously.

In legal theory, especially in the theory of law and economics, heuristics are used when case-by-case analysis would be impractical, insofar as "practicality" is defined by the interests of a governing body (Gerd and Christopher 2007). For instance, in many states in the United States the legal drinking age is 21, because it is argued that people need to be mature enough to make decisions involving the risks of alcohol consumption. However, assuming people

mature at different rates, the specific age of 21 would be too late for some and too early for others. In this case, the somewhat arbitrary deadline is used because it is impossible or impractical to tell whether one individual is mature enough that society can trust them with that kind of responsibility. Some proposed changes, however, have included the completion of an alcohol education course rather than the attainment of 21 years of age as the criterion for legal alcohol possession. This would situate youth alcohol policy more on a case-by-case model and less on a heuristic one, since the completion of such a course would presumably be voluntary and not uniform across the population.

In computer science, a heuristic is a technique designed to solve a problem that ignores whether the solution can be proven to be correct, but which usually produces a good solution or solves a simpler problem that contains or intersects with the solution of the more complex problem. Newell, & Simon, (1976) discuss the Heuristic Search Hypothesis as a physical symbol system which will repeatedly generate and modify known symbol structures until the created structure matches the solution structure. That is, each successive iteration depend upon the step before it, thus the heuristic search learns what avenues to pursue and which ones to disregard by measuring how close the current iteration is to the solution. Therefore, some possibilities will never be generated as they are measured to be less likely to complete the solution. A heuristic method can accomplish its task by utilizing search trees. However, instead of generating all possible solution branches, a heuristic selects branches more likely to produce outcomes than other branches. It is selective at each decision point; picking branches that are more likely to produce solutions. By using heuristics, time can be reduced when solving problems.

Teaching and learning of mathematics are complex processes. This is because each student is an individual with a unique personality. Students acquire knowledge, skills and attitudes at different times, rates and ways. The teachers' teaching techniques, though right may not be appropriate to all topics, for all students and at all times. This calls for an improved generalized approach of teaching mathematics. Heuristic strategy as an innovative approach in teaching and learning is the self-invitation and self-directed development of the learner (Blinkston, 2000). It is an experience based technique that helps in the problem solving, learning and discovery that is characterized by questioning hints, outlining of strategies and starting solution. An overview of heuristic instructional model holds its root to the right signal a student acquires from his immediate environment. Educationalist and Psychologist have been working on this natural way of learning owing to the shortcomings of the conventional teaching methods

(Obodo, 1997). The International Council of the Association of Science Education (ICASE, 2013), described the failure of conventional approaches and advocated that innovative approaches help students learn and think logically. Kimpley, (2006) specifically emphasized not only the acquisition of knowledge, but also problem solving and decision making skills. This may address the urgent need for improvement in Science, Technology and Mathematics (STM) education for sustainable development. With the increasing development in the world of science and technology, the science educators are further challenged with the task of producing competent manpower in the field of science and engineering. This imposes great demand on mathematics teachers to devise appropriate instructional strategies that could deliver the right candidates to face the emerging challenges at the tertiary level.

The need to find alternative strategies for improving the performance of students in mathematics has stimulated a number of pedagogical claims and counter claims (Stantrock 2004; Kahneman and Shane 2002, Xu Ching et al 2008, Abonyi, 2013). While Kahneman and Shane argue that heuristics is based on discovery learning and will enhance mastery of the concept in linear algebra, Santrock (2004) argued that heuristics are merely "rules of thumb" and educated guesses based on common sense. Xu Chung (2008) in his argument took a middle course. While he could not dismiss the heuristics as a virile tool for teaching, he noted that approaches to learning which suggest a solution to a problem without actually ensuring its workability might be accompanied with errors. As an intervention to these raging controversies, and genuine quest for the most appropriate strategy for teaching linear algebra, this study is on a mission to provide empirical evidence on the efficacy of the heuristic approach on students' achievement in linear algebra.

1.2. Objectives of the Study

The major objective of this study is to determine the effects of heuristic instructional methods on students' achievement in linear algebra. Specifically the study:

1. Ascertained the effects of heuristic instructional method on students' achievement in algebra.
2. Ascertained the effects of heuristic instructional method on the mean achievement of male and female students in linear algebra.
3. Determined the interaction effect of methods and gender on students mean achievement in linear algebra.

1.3. Scope of the Study

The study was restricted to senior secondary schools (SSI). The study was conducted in Anambra State of Nigeria. In terms of content scope the following topics were taught:

1. 1st and 2nd laws of indices
2. 3rd and 4th laws of indices
3. Identical equations

1.4. Research Questions

The following research questions guided the study:

1. What is the effect of heuristic method of teaching on students' mean achievement in algebra?

2. What is the effect of heuristic method of teaching on the mean achievement scores of males and females in algebra?

3. What is the interaction effect of method and gender on students mean achievement scores in linear algebra?

1.5. Hypotheses

The following null hypotheses were tested at an alpha level of 0.05:

HO_1: There is no significant difference in the mean achievement scores of students taught algebra using the heuristic method and those taught using the conventional approach.

Ho_2: There is no significant difference in the mean achievement scores of male and female students' taught algebra using the heuristic method.

HO_3: The interaction effect of method and gender on the mean achievement scores of students in algebra will not be significant.

2.1. Methodology

This study adopted the quasi-experimental design. Specifically this study employed Pretest Posttest non-equivalent control group design. Two secondary schools were drawn for the study through simple random sampling technique. Out of the two schools, one was assigned to the treatment group while the other was assigned to the control group through a simple toss of coin. The instrument used for data collection is Algebra Achievement Test. The instrument is a multiple-choice test with a total of thirty items. The items were drawn from the units that were taught during the experiment. The instrument was subjected to both face and content validation. The face validation scrutinized the items in terms of relevance, general test format, suitability and clarity. After the face validation and modification in line with recommendations of the specialists the instrument was further subjected to content validation using a standardized test blueprint. The Algebra Achievement Test was assessed for reliability using the Kuder-Richardson's (K-R20) approach. The test of

internal consistency with the K.R 20 yielded a reliability index of 0.60.

Two instructional packages were used for this study. The first package is the heuristic based instructional package while the second is the conventional instructional package. The heuristic package is identical to the conventional package in terms of content, basic instructional objectives and mode of evaluation. The only difference is in the instructional activities where heuristic package deviates from the conventional approach by employing heuristic procedures during the instructional process. The heuristic based package was used for the treatment group whereas the conventional package was used for the control group.

At the onset of the experiment, subjects in both treatment and control groups were given the pre-test. After the pre-test the regular mathematics teachers started the experiment in their respective schools adhering strictly to the lesson procedure that was developed from the packages during the pre-experimental conference. The experiment was conducted during the normal school periods, following the normal timetable of the school. At the end of the experiment that lasted for eight weeks the teachers administered the post-tests to the subjects in the two groups. Data were collected from the pre-test and post-test on the achievement of the students. These were kept separately for the two groups and were used to answer the research questions and test the null hypotheses. Research questions were answered using mean and standard deviation while hypotheses were tested at an alpha level of 0.05 using the Analysis of Co-Variance (ANCOVA)

2.2. Control of Extraneous Variables

The following procedures were adopted by the researcher to ensure that extraneous variables, which might influence the internal validity of the findings, were controlled:

Teacher Variable

In order to minimize errors, which might arise as a result of teacher difference, the researcher organized a pre-experimental training for the mathematics teachers that were used for the study. Separate training was organized for teachers in the two groups. The essence of the training is to help establish a uniform instructional standard among the mathematics teachers. All the topics for this study were treated during the training so that the mathematics teachers in each group could adopt a uniform approach. A manual containing the specifications of the packages was made available to the teachers that participated in the experiment. In addition the researcher monitored the experiment very closely so as to ensure that no teacher deviated from the agreed format.

Intergroup Variable

Because intact classes were used for this study it is implied that initial equivalence was not achieved for the research subjects in the two groups. In order to eliminate the errors of non-equivalence arising from the non-randomization of the subjects, the researcher used the analysis of co-variance (ANCOVA) for data analysis.

Subject Interaction

The researcher did not select treatment and control group from the same school to ensure that the students in the treatment and control groups do not mix up at all. This was to reduce the errors arising from interaction and exchange of ideas among research subjects from the two groups and further eliminate the possibility of a John Henry Effect.

3. Results
3.1. Research Question
Research Question 1:

What is the effect of heuristic method of teaching on students' mean achievement scores in mathematics?

To answer this research question, mean scores and standard deviation of students taught algebra with heuristic and conventional methods were computed and presented in Table 1

Table 1: *Mean and standard deviation of students' achievement for the two groups in Linear Algebra*

Groups	N	Mean	Standard dev.
Heuristic Approach	97	64.979	12.557
Conventional Approach	102	52.108	13.467

Table 1 shows that students taught with heuristic approach had a mean score of 64.98 while students taught the same linear algebra with conventional approach had a mean score of 52.11. This shows that heuristic method is superior to the conventional method in fostering students' achievement in linear algebra.

Research Question 2

What is the effect of Heuristic method of teaching on the mean achievement scores of male and female students in linear algebra?

To answer this research question, the mean scores and standard deviations of male and female students taught linear algebra with heuristic method of teaching were computed as shown in table 2.

Table 2: *Mean and standard deviation of male and females taught with Heuristic Method*

Students	Mean	Standard deviation	N
Male	67.32	13.56	56
Female	61.78	10.37	41

In table 2 the male students taught linear algebra with heuristic method had mean score of 67.32 with standard

deviation of 13.56 while the females had mean score of 61.78 with standard deviation of 10.37. The male students taught linear algebra with heuristic method performed better than their female counterparts taught the same topics with heuristic method.

Research Question 3

What is the interaction effect of the methods and genders on the students' mean achievement scores in linear algebra?

Mean scores of male and female students taught algebra using heuristic method and those taught with the conventional method were presented and used to x-ray interactions.

Table 3: *Interaction effect of gender and method on students' achievement in linear algebra*

Method	Gender	
	Male	Female
Heuristic method	67.32	61.78
Conventional method	55.27	49.61

As shown in Table 3 the heuristic method is superior to the conventional method at the two levels of gender. This implies that there is no interaction between method and gender on students mean achievement in linear algebra

3.2. Hypotheses
The Hypotheses were tested at 0.05 level of significance.

H_{O1}: *There is no significant difference in the mean achievement scores of students taught linear algebra using the heuristic method and those taught the same linear algebra using conventional method of teaching.*

H_{O3}: *The Interaction effect of method and gender on the mean achievement scores of students in mathematics will not be significant.*

Hypotheses 1 and 3 were tested using analysis of covariance (ANCOVA). The summary is presented in table 4.

Table 4: *Analysis of Co Variance for students overall linear algebra achievement scores by teaching methods and by gender*

Source of variation	Sum of squares	DF	Mean square	F cal	t. Critical
Covariates	16687.98	1	16687.98	218.96	
Pre-test	16687.98	1	16687.98	218.96	
Main Effects	10198.97	2	5099.48	66.910	
Method	8403.544	1	8403.54	110.26	3.86
Gender	849.918	1	849.91	11.15	
2-way Interactions	20.39	1	20.39	.268	3.86

(Method and Gender)				
Explained	26907.35	4	6726.839	88.26
Residual	14785.62	194	76.21	
Total	41692.9	198	210.57	

For hypothesis 1, the ANCOVA table shows that the F-cal (110.262) is greater than the critical value (3.86) at an alpha level of 0.05. The decision rule is to reject the null hypothesis when the calculated value exceeds the critical value at a given probability level. Since the calculated value is greater than the critical value, the null hypothesis was rejected. The researcher, therefore, concludes that there is a significant difference in the mean achievement scores of students taught linear Algebra using the heuristic approach and those taught linear algebra using the conventional approach.

For hypothesis 3, result in table 4 reveals that for two way interaction, the F-cal is .268 while the critical value at 95% confidence level is 3.86. Based on the decision rule, the researcher upholds the null hypothesis and concludes that there is no significant interaction between gender and instructional approach on students' mean achievement in linear algebra.

Ho₂: *There is no significant difference in the mean achievement scores of male and female students taught linear algebra using the heuristic approach.*

Male and female students in the treatment group only were used to test this hypothesis. This hypothesis was also tested using the Analysis of Co-variance. Summary of result is shown in Table 5

Table 5: *analysis of Co-variance of mean achievement scores of male and female students taught linear algebra using heuristic method.*

Source of variation	Sum of squares	DF	Mean square	F cal	F critical
Covariates	6996.674	1	699.674	86.724	
Pre-test	6996.674	1	699.674	86.724	
Main Effects	557.602	1	557.602	6.911	
Gender	557.602	1	557.602	6.911	3.96
Explained	7554.276	2	3777.138	46.818	
Residual	7583.682	94	80.677		
Total	15137.959	96	157.687		

For hypothesis 2, table 5 reveal that F-cal (6.911) is greater than the critical value (3.86) at alpha level of 0.05. Since the

calculated value is greater than the critical value at the given alpha level, the null hypothesis is rejected. The researcher, therefore, rejects the null hypothesis and concludes that there is a significance difference between the mean achievement score of male and female students taught linear algebra using the heuristic approach.

4. Discussion of Results

The results obtained from this study revealed that the students taught linear algebra with heuristic method of teaching performed better than the students taught using the conventional method. The success of the experimental group may be attributed to the anchoring effects that heuristic method has on learning. The heuristic method provides the potential of helping learners to conceptualize science as being transparent and the concepts as being interrelated. The difficulty and low enrolment in mathematics in school certificate examination is linked to the difficulty in understanding of mathematical concepts. The heuristic method makes students' understanding of the concepts to be easier and subsequently improve upon their achievement scores. The findings of this study are not surprising but understandable. According to Ngozi (2003), new innovation in teaching and learning attracts attention of the learners, motivates them, reduces the abstract nature of the subject and facilitates recall of learnt information.

The result of the study also showed a significant difference between the mean achievement scores of male and female students taught mathematics using heuristic method. The heuristic method of teaching favours the males more than the females. In addition the result of the study showed that there is no significant interaction between method and gender on achievement of males and females in linear algebra. Summary of results indicate that heuristic approach is superior to the conventional approach at the two levels of gender in fostering achievement. Treatment interaction, according to Abonyi (2004) generally implies that different learners with different characteristics may profit more from one type of instructional than from another and that therefore it may be possible to find the best match of learners characteristics and instructional approach in other to maximize learning outcomes or whichever dependent variable that is involved. In this case there is no need for separation of instructional method for male and females since the heuristic method could be used successfully for the two groups.

5. Conclusion and Recommendations

From the results obtained in the investigation into the effect of heuristic method on students' achievement in linear algebra, the following conclusions were drawn.

1. Heuristic instructional approach fostered better achievement in linear algebra than the conventional

approach. Heuristic approach is therefore superior to conventional approach in facilitating students achievement in linear algebra..

2. The difference in the mean achievement scores of male and female students taught linear algebra using the using heuristic approach is significant

3. There is no significant interaction between gender and instructional approach on students' mean achievement in linear algebra.

Based on the findings of the study the researchers recommend the application of the heuristic approach in teaching mathematics both in primary and secondary schools. This will invariably motivate, stimulate and sustain students' interest in mathematics. Local, State and Federal Governments should encourage and sponsor in-service trainings, workshops and seminars so as to inculcate the rudiments of heuristic method in teaching mathematics in Secondary Schools. In addition the government, in conjunction with curriculum developers and mathematics teachers, should establish mathematics resource centers in each local government area of the state for the procurement and storage of relevant heuristic instructional materials. The federal, state government and professional bodies like the Science Teachers Associations should sensitize other stakeholder in the education sector on the efficacy of heuristic method.

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